2024

MATHEMATICS

Full Marks: 100

Pass Marks:33

Time : Three hours

Attempt all questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos.1-10, write the letter associated with the correct answer.

1. Let R be the relation in the set N given by R= $\{(a, b): a = b - 2,\ b > 6\}$.
Choose the correct answer. 1

A. (2, 4)∈ R

B. (3, 8)∈ R

C. (6, 8)∈ R

D. (8, 7)∈ R

2. $\cos^{-1}(\cos \frac{7\pi}{6})$ is equal to 1

A. $\frac{7\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

3. If A is an invertible matrix of order 2, then det (A^{-1}) is equal to 1

A. det(A)

B. $\frac{1}{\det(A)}$

C. 1

D. 0

4. If $x = a(\theta - sin\theta)$ and $y = a(1 - \cos\theta)$, then $\frac{dy}{dx}$ is equal to 1

A. $\sin(\frac{\theta}{2})$

B. $\cos(\frac{\theta}{2})$

C. $\cot(\frac{\theta}{2})$

D. $\tan(\frac{\theta}{2})$

5. A balloon, which always remains spherical has a variable radius. The rate at which its volume is increasing with the radius when the radius is 10 cm , is 1

A. $400\pi\ cm^3/\ cm$

B. $300\pi\ cm^3/\ cm$

C. $200\pi\ cm^3/\ cm$

D. $100\pi\ cm^3/\ cm$

6. $\int \frac{dx}{x^2+2x+2}$ equals 1

A. $x\ \tan^{-1}(x+1) + C$

B. $\tan^{-1}(x+1) + C$

C. $(x+1)\ \tan^{-1} x + C$

D. $\tan^{-1} x + C$

7. The degree of the differential equation: $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is 1

A. 3

B. 2

C. 1

D. Not defined

8. Two events A and B will be independent , if 1

A. A and B are mutually exclusive

B. $P(A) = P(B)$

C. $P(A'B') = [1 - P(A)][1 - P(B)]$

D. $P(A) + P(B) = 1$

9. If α, β and γ are direction angles of a line , then the value of

$sin^2\alpha + sin^2\beta + sin^2\gamma$ is 1

A. 1

B. −1

C. 2

D. −2

10. The unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1,2,3) and (4,5,6) respectively is 1

A. $\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{2}{\sqrt{3}}\hat{k}$

B. $\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$

C. $\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{5}}\hat{\jmath} + \frac{1}{\sqrt{7}}\hat{k}$

D. $\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} - \frac{2}{\sqrt{3}}\hat{k}$

11. Define diagonal matrix. 1

12. If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, then find AB. 1

13. Define continuity of a function $f(x)$ at $x = c$. 1

14. Prove that the function $f(x) = \log sinx$ is increasing on $(0, \frac{\pi}{2})$. 1

15. Write the value of $\int \sec x \, dx$. 1

16. Evaluate $\int e^x secx \, (1 + tanx)dx$. 1

17. Evaluate : $\int \frac{\sin x}{1+\cos x} \, dx$ 1

18. Write the definition of scalar product of two non zero vectors. 1

19. Solve the differential equation $\frac{dy}{dx} - y = 1; (y \neq -1)$ 1

20. Write the multiplication rule of probability for two events of a sample space. 1

ontd.

21. Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}(-\frac{1}{2})$ 2

22. If $\tan^{-1}\left(\frac{3}{4}\right) = x$, find the values of $\sin x$ and $\cos x$. 2

23. Find $\frac{dy}{dx}$, if $sin^2x + cos^2y = 1$. 2

24. Differentiate x^x with respect to x. 2

25. Verify that the function $y = a\ sinx + bcos\ x$,

(where a and b are constants) is a solution of the differential

equation $\frac{d^2y}{dx^2} + y = 0$ 2

26. If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$,then prove that 2

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

27. Find the values of p so that the lines

$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. 2

28. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that the youngest is a girl? 2

29. Show that the relation R in the set $A = \{x \in \mathbf{Z}: 0 \leq x \leq 12\}$, given by R=$\{(a, b): |a - b| is\ a\ multiple\ of\ 4\}$ is an equivalence relation. 4

Or

Let $A = \mathbf{R} - \{3\}$ and $B = \mathbf{R} - \{1\}$.Prove that the function $f: A \to B$ defined by $f(x) = (\frac{x-2}{x-3})$ is one-one and onto.

30. For any square matrix A with real number entries, prove that A+A' is a symmetric matrix and A−A' is a skew symmetric matrix. 4

Or

Prove that a square matrix A is invertible if and only if A is non-singular matrix.

31. If $y = (\tan^{-1} x)^2$, show that $(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$. 4

Or

For what values of a and b is the function $f(x) = \begin{cases} x^2, x \le c \\ ax+b, x > c \end{cases}$

differentiable at $x = c$?

32. Prove that $\int \sqrt{a^2+x^2}\,dx = \frac{1}{2}x\sqrt{a^2+x^2} + \frac{a^2}{2}\log|x+\sqrt{a^2+x^2}| + C$ 4

Or

Prove that $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, if\ f(2a-x) = f(x) \\ 0, \quad if\ f(2a-x) = -f(x) \end{cases}$

33. Find the particular solution of the differential equation 4

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}\ ; y = 0 \text{ when } x = 1$$

Or

Find the particular solution of the differential equation

$x^2dy + (xy + y^2)dx = 0$ given that $y = 1$ when $x = 1$

34. Using integration, find the area of the region bounded

by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

4

35. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle.

4

36. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$. 6

Hence find A^{-1}.

37. The sum of the perimeter of a circle and a square is k, where k is some constant. Prove that the sum of their areas is least when the side of the square is double the radius of the circle. 6

Or

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

38. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{cos^2x}{cos^2x+4sin^2x} dx$ 6

Or

Prove that $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}\, dx = \frac{1}{20}\, log\, 3$

39. Derive the vector equation of a line passing through a given point and parallel to a given vector, and hence obtain the Cartesian equation of the line. 6

Or

Define skew lines. Find the shortest distance between two skew lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$

40. Define objective function and constraints of a Linear Programming Problems.

Solve the linear programming problem graphically(Don't use graph paper) 6

Minimise $Z = 200x + 500y$

Subject to

$x + 2y \geq 10$

$3x + 4y \leq 24$

$x \geq 0, y \geq 0$

41. An urn contains 5 balls. 2 balls are drawn and found to be white. What is the probability that all the 5 balls are white? 6

Or

In a test , an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question , given that he correctly answered it.

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