# 2018

# MATHEMATICS

**Full Marks : 100**

**Pass Marks : 33**

**Time : Three hours**

*Attempt all Questions.*

*The figures in the right margin indicate full marks for the questions.*

*For Question Nos. 1 – 6, write the letter associated with the correct answer.*

1. If $f$ and $g$ are functions defined from R to R by $f(x) = (x+1)^2$ and $g(x) = x^2 + 1$ then the value of $(f \circ g)(-3)$ is

*A.* 17

*B.* 121

*C.* 44

*D.* 40 1

2. The value of $Sin^{-1}\frac{1}{3} - Cos^{-1}\left(-\frac{1}{3}\right)$ is

*A.* $-\frac{\pi}{2}$

*B.* $\frac{\pi}{2}$

*C.* $\frac{\pi}{3}$

*D.* $-\frac{\pi}{3}$ 1

3. If $A=\begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is such that $A^2 = I$, then

*A.* $1 + a^2 + bc = 0$

*B.* $1 - a^2 + bc = 0$

*C.* $1 - a^2 - bc = 0$

*D.* $1 + a^2 - bc = 0$ 1

4. The volume of a spherical balloon is increasing at the rate of 25 $cm^3$/sec. The rate of increase of its surface area when its radius is 5cm is

*A.* 5 $cm^2$/sec.

*B.* 10 $cm^2$/sec.

*C.* 15 $cm^2$/sec.

*D.* 20 $cm^2$/sec. 1

5. The value of $\int \frac{xdx}{4+x^4}$ is

*A.* $\frac{1}{4}\tan^{-1}\left(x^2\right)+C$

*B.* $\frac{1}{4}\tan^{-1}\left(\frac{x^2}{2}\right)+C$

*C.* $\frac{1}{2}\tan^{-1}\left(x^2\right)+C$

*D.* $\frac{1}{2}\tan^{-1}\left(\frac{x^2}{2}\right)+C$ 1

6. The line $\frac{x-2}{3}=\frac{y-2}{4}=\frac{z-4}{5}$ is parallel to the plane

A. $2x+3y+4z=0$

B. $3x+4y+5z=7$

C. $x+y+z=2$

D. $2x+y-2z=0$ 1

7. Define identity element for a binary operation defined on a set. 1

8. Write the value of $\sin\left(\tan^{-1}\frac{3}{4}\right)$. 1

9. Define identity matrix. 1

10. If A is a matrix of order $2\times 3$ and B is a matrix such that $A'B$ and $BA'$ both are defined, then what is the order of B ? 1

11. State Lagrange's mean value theorem. 1

12. Define increasing function. 1

13. Write the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x dx$. 1

14. What is meant by solution of a differential equation ? 1

15. Define vector product of two vectors. 1

16. For a binomial probability distribution with parameter $n, p$ and $p+q=1$, what are the mean and variance ? 1

17. Let Z be the set of integers. Show that the relation R={ (a,b):a,b ∈ Z and a + b is even} is an equivalence relation. 3

18. Show that $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$. 3

19. Find $\frac{dy}{dx}$, if $x^y \cdot y^x = 1$. 3

20. Write the steps for solving a homogeneous differential equation. 3

21. If $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$,

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then prove that $\bar{a}\cdot(\bar{b}\times\bar{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$. 3

22. A family has two children. Find the probability that both the children are boys if it is known that at least one of the children is a boy. 3

23. Prove that every square matrix is uniquely expressible as the sum of a symmetric matrix and a skew-symmetric matrix. 4

24. Using integration, find the area of the smaller region bounded by the curve $x^2 + y^2 = 1$ and the line $x + y = 1$. 4

25. Solve the differential equation $\left(1+x^2\right)\frac{dy}{dx} + y = e^{\tan^{-1}x}$, given that $y = 1$ when $x = 0$. 4

26. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors then prove that angle between $\vec{a}$ and $\vec{a}+\vec{b}+\vec{c}$ is $cos^{-1}\frac{1}{\sqrt{3}}$. 4

***OR***

Using vector, find the area of the triangle whose vertices are A (3, –1, 2), B (1, –1, –3) and C (4, –3, 1).

27. Find the distance of a point A with position vector $\vec{a}$ from the plane $\vec{r}\cdot\vec{n} = q$. 4

28. Find the shortest distance between the following pairs of parallel line $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ and $\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + \mu\left(4\hat{i} + 6\hat{j} + 8\hat{k}\right)$. 4

***OR***

Find the equation of the plane through the point (1, 0, –1) and (3, 2, 2) and parallel to the line $x-1 = \frac{y-1}{-2} = \frac{z-2}{3}$.

29. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & -2 & -1 \end{bmatrix}$, find $A^{-1}$. Using $A^{-1}$ solve the system of linear equations:

$$2x + y + z = 3,\ 2x + z = 5,\ -2y - z = 1.$$ 6

30. Prove that the function defined as 6

$$f(x) = \begin{cases} x \cdot \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, & \text{if } x \neq 0, \\ 0 & \text{if } x = 0 \end{cases}$$

is continuous but not derivable at $x = 0$.

31. Show that the volume of the largest cone that can be inscribed in a given sphere is $\frac{8}{27}$ of the volume of the sphere. 6

***OR***

A window is in the form of a rectangle surrounded by a semi-circular opening. The total perimeter of the window is 10cm. Find the dimensions of the window to allow maximum light through the opening.

32. State any six properties of definite intergrals. 6

33. Evaluate $\int \frac{1}{x^3+1}\, dx$ 6

***OR***

Evaluate $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\, dx$

34. In a school scholarship is to be given to the students of XI and XII Classes. It is decided that at least 5 students from XI Class and at least 4 students from XII Class should get the scholarship. Each scholarship holder of XI Class will get Rs. 300 per month and each scholarship holder of XII Class will get Rs. 400 per month. The total number of scholarship holder should not be less than 10 but should not be greater than 15. How many students of each class be selected so as to maximise the amount of scholarship ? Solve the LPP graphically. (Graph paper will not be supplied). 6

***OR***

A manufacturer makes two types of cups A and B. Three machines are required to manufacture the cups and the time in minutes required by each is as given below :

| **Type of cup** | **Machine** | | |
|---|---|---|---|
| | I | II | III |
| A | 12 | 18 | 6 |
| B | 6 | 0 | 9 |

Each machine is available for a maximum period of 6 hours per day. The profit on each cup A is Rs. 7·50 and on B it is Rs. 5·00. How many cups of each type should be manufactured in a day to maximise the profit? Solve the LPP graphically. (Graph paper will not be supplied).

35. A girl throws a dice. If she gets a 5 or 6, she toses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin two times and notes the number of heads obtained. If she obtained exactly two heads, what is the probability that she throws 1, 2, 3 or 4 with the dice ? 6